ST740 – Assignment 1 – Due 9/7

The homework is due in class on Wednesday, 9/7. Show work for questions that say "derive" and include code used to produce the results. You many work in groups, but write up separate solutions.

1. Assume $Y_1, ..., Y_n|\theta \sim \text{Uniform}(0, \theta)$ independent over i.
   (a) Identify a conjugate family of prior distributions for θ and derive the posterior
   (b) Now assume you observe $n = 50$ samples as below

   ```
   > set.seed(919)
   > Y <- runif(50,0,10)
   > range(Y)
   [1] 0.05161189 9.75337425
   ```

   Use an uninformative prior and summarize the posterior in a table and plot
   (c) Is the posterior senstive to the prior?
   (d) What is the posterior predictive probability that Y_{n+1} will be a new record, i.e.,

   $$\text{Prob}(Y_{n+1} > \max\{Y_1, ..., Y_n\}|Y_1, ..., Y_n)$$

   (e) Why is (d) not exactly $1/(n + 1)$, or is it?

2. Download the daily weather data from RDU Airport

   ```
   file  <- "https://www4.stat.ncsu.edu/~bjreich/ST740/RDU.csv"
   dat   <- read.csv(url(file))
   TMAX  <- dat[,2]/10
   TMIN  <- dat[,3]/10
   MONTH <- dat[,4]
   ```

   (a) Plot the sample correlation between daily minimum (TMIN) and maximim (TMAX) temperature by month
   (b) Let $Y = (T_{MIN}, T_{MAX})$ be a bivariate response and fit the model $Y|\Sigma \sim \text{Normal}(\bar{Y}, \Sigma)$ where $\bar{Y}$ is the sample mean of Y (say it is fixed and known) and Σ is the unknown 2×2 covariance matrix. Specify a conjugate family of prior distributions for Σ and derive the corresponding posterior.[1]
   (c) Select an uninformative prior distribution and summarize the induced prior distribution on the correlation $\rho = \Sigma_{12}/\sqrt{\Sigma_{11}\Sigma_{22}}$ in figure.
   (d) Fit the model to the temperature data separately by month (including a monthly $\bar{Y}$) and plot the posterior distribution of the correlation between T_{MIN} and T_{MAX} (ρ) by month. Is there a statistically significant correlation between these variables? Are there statistically signficant differences by month?

[1] Recall that for vector a and square matrices B and C that $a^T Ba = \text{trace}(a^T Ba) = \text{trace}(Baa^T)$ and $\text{trace}(a^T B) + \text{trace}(a^T C) = \text{trace}(a^T (B + C))$