ST740 – Assignment 3 – Due 10/5

Show work for questions that say "derive" and include code used to produce the results. You many work in groups, but write up separate solutions.

For this assignment, you will analyze the streamflow data at

```
file  <- "https://www4.stat.ncsu.edu/~bjreich/ST740/HCDN_annual_max.RData"
load(url(file))
```

These data are collected as part of the USGS' Hydro-Climatic Data Network of sites that have not been affected by human development for the purpose of studying the effects of climate change on flood risk. This workspace includes annual maximum streamflow (cubic feet per second) at 702 locations from 1950-2021 in the matrix `Y`. The spatial coordinates of the stations are given in the matrix `s` and the years are given in the vector `year`. Let Y_{st} be the observation at location $s \in \{1, ..., 702\}$ and time (year) $t \in \{1, ..., 72\}$ and $X_t = (year_t - 1985)/10$ be a linear trend covariate, where $year_t$ is the year for observation t. Discard sites with missing observations, which leaves 236 sites. The goal is to determine if and where the distribution of extreme streamflow has changed over the past 72 years.

Let $Z_{st} = \log(Y_{st} + 1)$ and assume the linear model $Z_{st}|\beta_s \sim \text{Normal}(\beta_{0s} + X_t\beta_{1s}, \sigma^2)$, independent over s and t. The random coefficients are modelled as $\beta_s = (\beta_{0s}, \beta_{1s})^T \sim \text{Normal}(\mu, \Sigma)$ where the 2×1 mean vector μ, 2×2 covariance matrix Σ and error variance σ^2 have uninformative conjugate priors.

1. Specify and defend your prior distribution.

2. Derive the full conditional distributions of β_s, μ, Σ and σ^2.

3. Write a Gibbs sampler to fit this model and give trace plots of the elements of β_1, σ^2, μ and Σ to show it has converged.

4. Make maps of the posterior mean and standard deviation of β_{1s}, and the posterior probability that $\beta_{1s} > 0$. Discuss the changes over time in extreme streamflow.